

1. The weekly demand for wigits is given by

$$p = 800 - 5x \quad (0 \leq x \leq 160)$$

where p denotes the unit price and x denotes the quantity demanded.

The weekly total cost function associated with manufacturing wigits is given by

$$C(x) = 800 + 600x - 3x^2$$

Find:

a: the revenue function: *Revenue is the quantity sold, x times the price at which it is sold, $p=800-5x$ so:*

$$\mathbf{R(x) = 800x - 5x^2}$$

b: the profit function: *Profit is revenue - cost or $R(x) - C(x)$ so (make sure you distribute the - sign over all the components of cost):*

$$\mathbf{P(x) = R(x) - C(x) = 800x - 5x^2 - (800 + 600x - 3x^2) = 800x - 5x^2 - 800 - 600x + 3x^2 = 200x - 2x^2 - 800}$$

c: The marginal cost function: *is just the derivative of the cost function:*

$$\mathbf{C'(x) = 600 - 6x}$$

d: The marginal revenue function: *is just the derivative of the revenue function:*

$$\mathbf{R'(x) = 800 - 10x}$$

e: The marginal profit function: *is just the derivative of the profit function:*

$$\mathbf{P'(x) = 200 - 4x ; Which is also R'(x) - C'(x)}$$

f. Compute values of all five of these functions if the quantity demanded is 50: *Just plug in 50 to the above to get:*

$$\mathbf{R(50) = 800(50) - 5(50)^2 = 40,000 - 12,500 = 27,500}$$

$$\mathbf{P(50) = 200(50) - 2(50)^2 - 800 = 10,000 - 5000 - 800 = 4200}$$

$$\mathbf{C'(50) = 600 - 6x(50) = 300}$$

$$\mathbf{R'(50) = 800 - 10(50) = 300}$$

$$\mathbf{P'(x) = 200 - 4(50) = 0}$$

g. Interpret your results:

Since $P'(x) = 0$, you cannot increase profit by increasing production, which you could if $P'(x) > 0$. Nor can you increase profit by decreasing production, which you could if $P'(x) < 0$. So your profit is maximized when you sell 50 wigits.

2. Use Differentials to approximate the square root of 5.

We know $\sqrt{4} = 2$ and that is the closest perfect square to 5.

So we let $x=4$; $dx = 1$ (so $x+dx = 5$)

and find dy which will approximate the change in \sqrt{x} as x moves from 4 to 5,

using $dy = f'(x) dx$, with $f(x) = \sqrt{x}$.

$f'(x) = \frac{1}{2} x^{-1/2} = 1/(2\sqrt{x})$; so $f'(4) = 1/4$. Thus $dy = (1/4)(1) = 1/4$ and $\sqrt{5} \approx \sqrt{4} + 1/4 = 2.25$

If the square root of 5 is 2.236, what is the error in your approximation?

2.25 - 2.236 = .014 is the error in this approximation.